QUANTUM MECHANICS AND BLACK HOLES

1. Introduction

An analysis of the dynamic behaviour of a particle within the gravitational field of a Schwarchild black hole $(S = 0, Q = 0)$, according to the conclusions of classical mechanics, predicts that it is impossible for a particle as a point mass with limited dimensions in space (X, Y, Z) to escape from a point which is located under the Schwarzchild radius which for a body of mass M is defined from the equation:

$$R_s = 2GM / c^2 \quad (1)$$

2. The classical approach

This is intepreted by the fact that the total energy E_0 of the particle of mass m and velocity u_o in a distance R_o (where $0 \le R_o < R_s$) from the center of mass of the black hole is negative ($E_0 < 0$).
Thus we have:

$$E_o = E_{K_o} - U_o < 0$$

or

$$1/2mu_o^2 < GMm / R_o \quad (2)$$

Inequation (2) states the fact that the particle will remain inside the gravitational field of the black hole for infinite time $(t = \infty)$.

3. The quantum mechanical approach

But if we study the phenomenon through the conlusions of quantum mechanics, we can observe that the particle has to face a barrier of gravitational potential energy of the general form $U(R)$ (i.e work $W(R)$) as it shown in diagram I.

It is known that a particle that has to face a square barrier of potential energy of height U $(U > E)$ and width δx has a probability P to pass through that barrier. This probability is given by:

$$P \cong e^{-2K\delta\chi} \quad (3)$$

where

$$K = \frac{\sqrt{2m(U - E)}}{\hbar} \quad (4)$$

Because the barrier of gravitational potential energy has the general shape $W(R)$ (as it shown in figure I) and assuming that it consists of n square barriers of width δR, the probability to pass through all that barriers is defined by:

$$P = e^{-2K_1\delta R} e^{-2K_2\delta R} ... e^{-2K_i\delta R} \quad (5) \quad \text{where } K_i = \frac{\sqrt{2m(W_i - E_{K_O})}}{\hbar} \quad (6)$$

Thus we have:

$$P = e^{-2(K_1 + K_2 + ...)\delta R} = e^{-2}\sum_i K_i \delta R \quad (7)$$

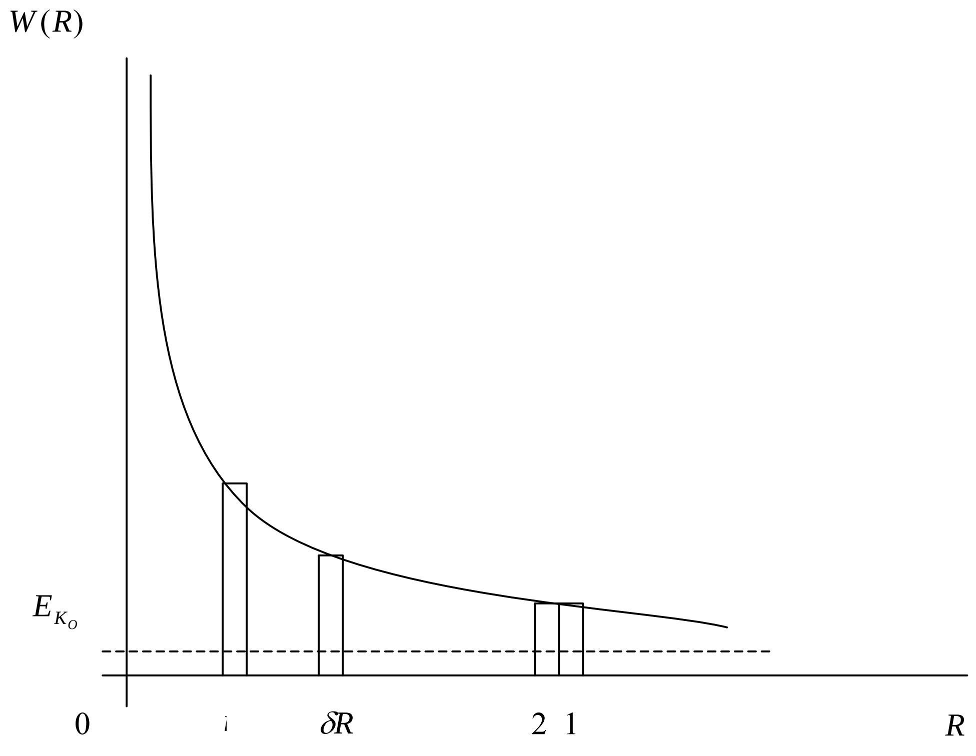
If we consider that $\delta R \to dR$ we get that:

$$P = e^{-2\int\sqrt{\frac{2m[W(R) - E_{K_O}]}{\hbar^2}}dR} \quad (8)$$

4. Conclusions

Relationship (8) defines mathematically the tunel effect according to which there is a probability P for the particle to escape to infinity (theoretically), by passing through the barrier of gravitational potential energy $W(R)$ that faces in point under the Schwarzchild radius.

Figure I



I. Particle of kinetic energy $E_{K_O} > 0$ inside a black hole, that faces a barrier of potential energy of height $W(R)$ has a probability P to pass through it.