QUANTUM MECHANICS AND BLACK HOLES

1. Introduction

An analysis of the dynamic behaviour of a particle within the gravitational field of a Schwarchild black hole \((S = 0, Q = 0)\), according to the conclusions of classical mechanics, predicts that it is impossible for a particle as a point mass with limited dimensions in space \((X, Y, Z)\) to escape from a point which is located under the Schwarchild radius which for a body of mass \(M\) is defined from the equation:

\[
R_s = \frac{2GM}{c^2}
\]

2. The classical approach

This is interpreted by the fact that the total energy \(E_0\) of the particle of mass \(m\) and velocity \(u_o\) in a distance \(R_o\) (where \(0 \leq R_o < R_s\)) from the center of mass of the black hole is negative \((E_0 < 0)\).

Thus we have:

\[
E_o = E_{K_o} - U_o < 0
\]

or

\[
\frac{1}{2}mu_o^2 < GMm / R_o(2)
\]

Inequation (2) states the fact that the particle will remain inside the gravitational field of the black hole for infinite time \((t = \infty)\).

3. The quantum mechanical approach

But if we study the phenomenon through the conclusions of quantum mechanics, we can observe that the particle has to face a barrier of gravitational potential energy of the general form \(U(R)\) (i.e work \(W(R)\)) as it shown in diagram I.
It is known that a particle that has to face a square barrier of potential energy of height $U$ ($U > E$) and width $\delta x$ has a probability $P$ to pass through that barrier. This probability is given by:

$$P \approx e^{-2K\delta x} \quad (3)$$

where

$$K = \frac{\sqrt{2m(U-E)}}{\hbar} \quad (4)$$

Because the barrier of gravitational potential energy has the general shape $W(R)$ (as it shown in figure I) and assuming that it consists of $n$ square barriers of width $\delta R$, the probability to pass through all that barriers is defined by:

$$P = e^{-2K_1\delta R} e^{-2K_2\delta R} \ldots e^{-2K_n\delta R} \quad (5)$$

where $K_i = \frac{\sqrt{2m(W_i-E_{K_i})}}{\hbar} \quad (6)$

Thus we have:

$$P = e^{-2(K_1+K_2+\ldots)\delta R} = e^{-2\sum_i K_i\delta R} \quad (7)$$

If we consider that $\delta R \to dR$ we get that:

$$P = e^{-2\int \frac{2m(W(R)-E_{K_0})}{\hbar^2} dR} \quad (8)$$
4. Conclusions

Relationship (8) defines mathematically the tunnel effect according to which there is a probability $P$ for the particle to escape to infinity (theoretically), by passing through the barrier of gravitational potential energy $W(R)$ that faces in point under the Schwarzschild radius.

Figure I

Particle of kinetic energy $E_{k_o} > 0$ inside a black hole, that faces a barrier of potential energy of height $W(R)$ has a probability $P$ to pass through it.